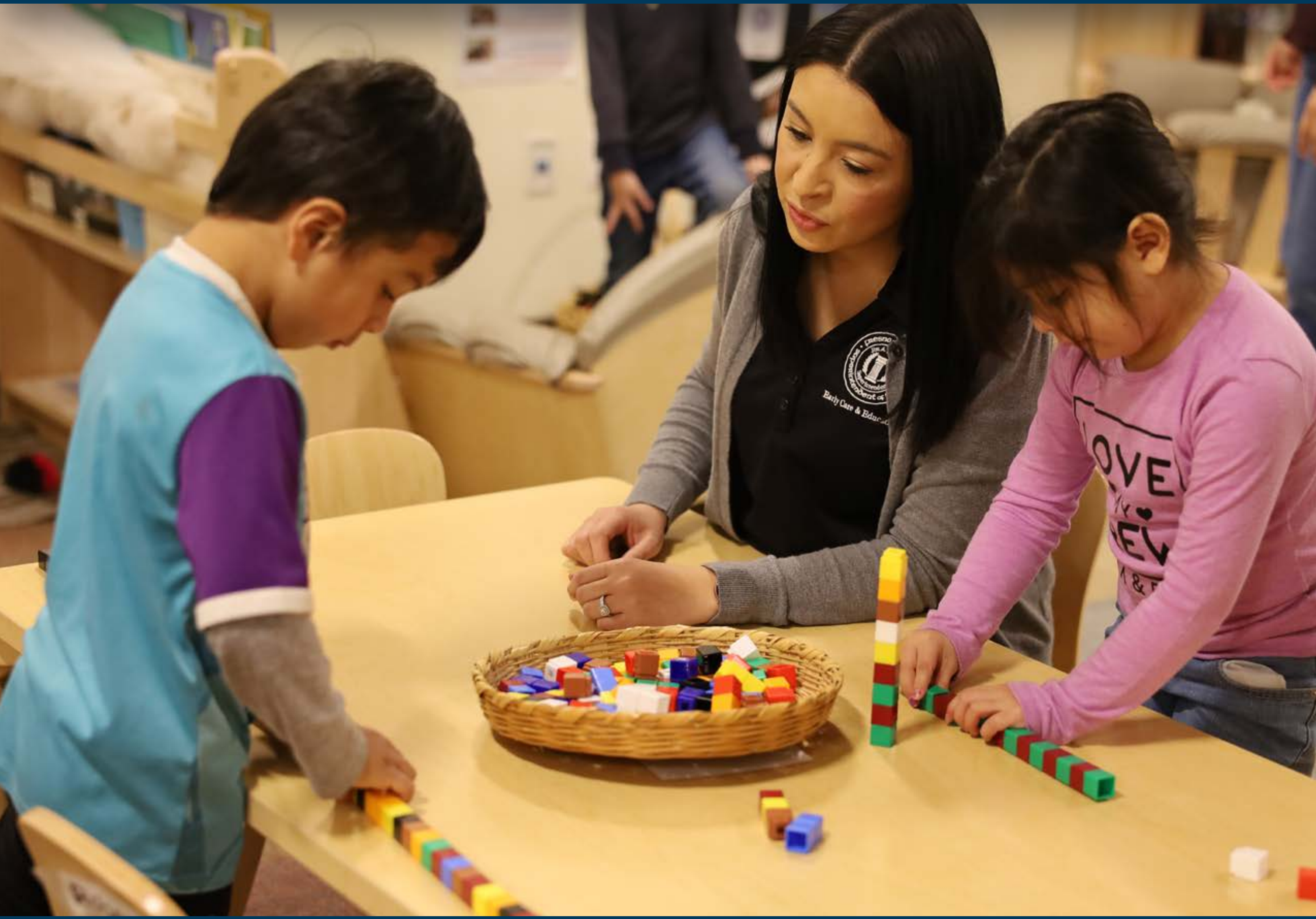
California Preschool/
Transitional Kindergarten

Learning Foundations

Mathematics



Universal PreKindergarten

For Three-to-Five-and-a-Half-Year-Old Children in Center-Based, Home-Based, and TK Settings

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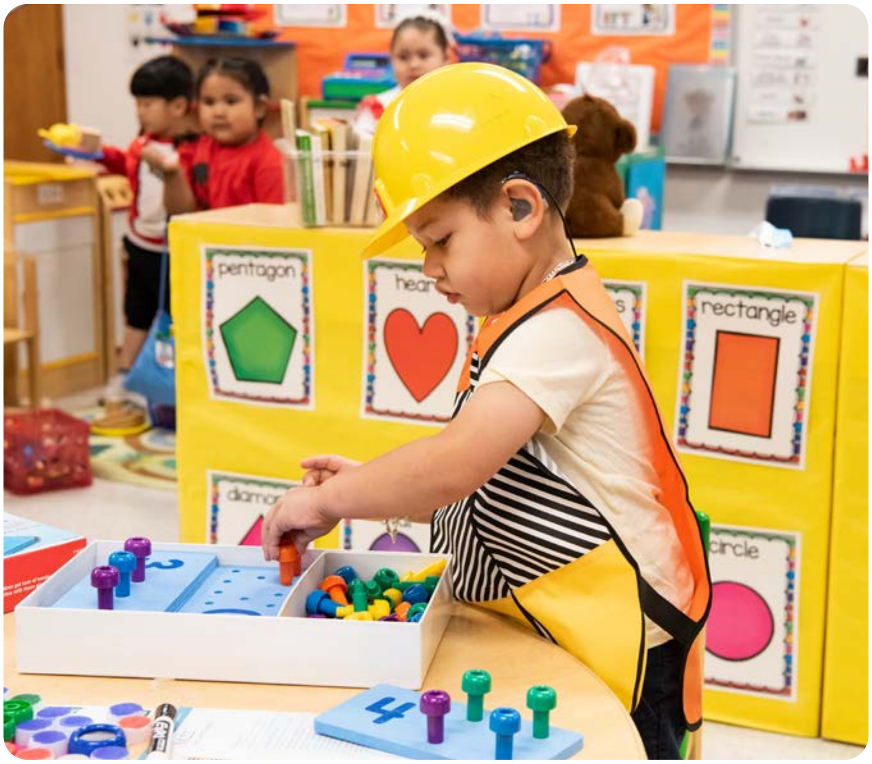
Introduction

Children are born with a natural ability for and interest in exploring mathematical concepts.[1] Through interactions with adults and their environment, children in the early years explore concepts such as quantity, shape, and size, and gain experience in counting, sorting, ordering, collecting data, and engaging in spatial thinking. Early mathematics skills are one of the most important factors in children's academic success.[2] The mathematical concepts and skills that young children develop set the foundation for their mathematics learning in the elementary years.[3]



The Preschool and Transitional Kindergarten Learning Foundations (PTKLF) provide guidance to all California early education programs, including transitional kindergarten (TK), federal and state preschool programs (for example, California State Preschool Program, Head Start), private preschool, and family child care homes, on the wide range of mathematics knowledge and skills that children age three to five and a half typically attain when attending a high-quality early education program. Teachers can use the PTKLF to guide their observations and set learning goals for children and plan for developmentally appropriate, equitable, inclusive practice, including how to design learning environments and create learning experiences that promote children's learning and development in the Mathematics domain. Early education programs can use the PTKLF to select and implement curricula aligned with the PTKLF, guide the selection of assessments aligned with the PTKLF, design and offer professional development and coaching programs for educators to support understanding and effective use of the PTKLF, and enhance preschool to third grade (P–3) continuity across learning goals and practice in mathematics.

Organization of Mathematics Domain

Strands and Sub-Strands

The PTKLF in Mathematics are organized into strands and sub-strands that address key areas in the development of knowledge and skills in mathematics.

- **Counting and Cardinality:** This strand describes children's ability to recite numbers, use **one-to-one-correspondence**, and understand the concept of **cardinality**. It also describes children's ability to identify the quantity of small groups of objects without counting (**subitizing**), compare quantities, and recognize **numerals**.
- **Operations and Algebraic Thinking:** This strand describes children's ability to solve simple arithmetic problems and reason about ideas such as adding to, putting together, taking apart, taking away, and sharing. Additionally, this strand describes children's abilities related to sorting or **classifying** objects by salient **attributes**.
- **Measurement and Data:** This strand describes young children's understanding of how objects can be compared, ordered, and measured by a variety of attributes, such as length, weight, or capacity. The foundations on data describe children's ability to collect, represent, and interpret data.
- **Geometry and Spatial Thinking:** This strand describes children's ability to identify, describe, and compare **two- and three-dimensional shapes** by noticing a shape's attributes (for example, points or sides). This strand also describes children's ability to identify positions and directions of objects in space using spatial vocabulary and mentally rotate, flip, or slide objects to solve problems (for example, rotating puzzle pieces).

The Mathematics domain is organized to align with the California Common Core State Standards in Mathematics.[4] This alignment allows teachers to draw connections between the knowledge, skills, and behaviors children build in their prekindergarten years and what they are expected to learn in kindergarten and beyond. In addition, the Mathematics foundations include Mathematical Practices, which describe the types of behaviors and dispositions that allow children to develop knowledge and skills in mathematics. The Mathematical Practices are identical to the Standards for Mathematical Practice listed in the California Common Core Standards but with added descriptions to provide teachers with context for how these practices apply to young children.

Foundations Statements

Within each sub-strand in the Mathematics domain are individual foundation statements that describe the competencies—the knowledge and skills—that children can be expected to demonstrate in a high-quality early education program. Children develop these competencies at different times and in different ways within their home, school, and community contexts. The foundation statements are intended to help teachers identify learning opportunities they can support.

Age Levels

Age-based foundation statements describe what children may often know and be able to do as a result of their experiences and unique developmental journey in mathematics. These statements are presented in two overlapping age ranges with full recognition that each child's development progresses over the early years with growth spurts and periods of skill consolidation in different domains at different points in time:

- An "Early Foundation" addresses knowledge and skills that children often demonstrate between three and four-and-a-half years of age.
- A "Later Foundation" addresses knowledge and skills that children often demonstrate between four and five-and-a-half years of age.

Examples

For each level of any given foundation, examples illustrate the diverse ways children may demonstrate their knowledge and skills. Examples across the Early and Later foundation levels show development over time. The first one or two examples in each foundation are aligned across the Early and Later age levels. Examples show how children may demonstrate a developing skill or knowledge as part of their everyday routines, learning experiences, and interactions with adults and peers. Examples also provide different ways in which children may demonstrate their developing skills in different contexts, whether indoors or outdoors, and in a range of activities throughout the day.

Multilingual learners possess foundational language abilities developed in the context of their relationships in their homes and communities. The use of their home language in the early education program serves as a powerful tool, supporting children's sense of belonging, bridging connections to their existing knowledge, and fostering deeper ties to their homes and communities. Examples in the home language of multilingual learners illustrate how multilingual children can further develop these foundational abilities by using their home language as part of their learning and daily interactions with peers and adults in the early education program. In instances where a teacher may not be fluent in a child's home language, various strategies can encourage multilingual learners to use their home languages, allowing them to leverage all of their linguistic capacities. To facilitate communication and understanding, the teacher can partner with staff or family volunteers who speak the child's home language. The teacher can also use interpreters and translation technology tools to communicate with families and gain insights about what a child knows and is able to do. All teachers should communicate with families about the benefits of bilingualism and how the home language serves as a critical foundation for English language development. Teachers should also encourage families to promote their child's continued development of the home language as an asset for overall learning.

Some examples include how the teacher may support children as they progress to the next level of development in the knowledge and skills of the foundation. Teachers may ask an open-ended question, scaffold learning by making a suggestion or giving a prompt, or comment on what a child is doing. The examples should help teachers gauge where a child's development is, consider how to support their development within their current skill level, and build toward the next skill level in that foundation. Furthermore, while the examples may provide teachers with valuable ideas for how to support children's learning and development as children build their knowledge or skill in mathematics, the examples are a small subset of all the different strategies teachers may employ to support children's learning and development in this domain. At the end of this introduction, the section How Teachers Can Support Children's Early Mathematics Learning offers ideas on ways to support children's learning and development in mathematics. Additionally, callout boxes with tips and strategies for teaching are embedded throughout the foundations to guide practice in the domain.

Diversity in Children's Early Mathematics Learning

Children enter early education programs with a range of mathematical experiences based on their home and community environments and family values related to mathematics. Traditions and routines in the home and community provide children with natural opportunities to develop their mathematical thinking. For example, rug weaving is an important practice in Native nations and tribal communities. Rug weaving requires creating intricate designs using various colors of yarn on a loom, exposing children to mathematical concepts like **patterns**, shapes, and measurement.[5]

Teachers should capitalize on the mathematical opportunities that are present in the child's home and community life and provide learning experiences that allow children to engage in mathematical thinking through cultural activities, traditions, and routines that are meaningful to the child.

Children's linguistic background can impact the way they learn and express mathematical ideas. Languages vary in how they represent mathematical concepts, including their number naming system. For example, languages such as Chinese or Japanese have a number system in which number words map directly onto a **base-ten** structure (for example, the word for "11" in Chinese is equivalent to "ten-one"); as such, the meaning of number words past 10 is obvious. Languages such as English and Spanish have a less obvious number word system. Some of the number words in these languages do not map clearly onto a base-ten structure (for example, the word for "11" in English is "eleven"). These varying number word systems can impact how children learn to count. Research suggests that children learning languages with more transparent number systems, such as Chinese, Japanese, or Arabic, may have an easier time—at least at first—in learning to recite numbers, especially numbers that follow 10.[6]

Multilingual learners may learn and communicate mathematical ideas in more than one language. However, the ease with which young children express their mathematical knowledge in their various languages depends on the amount of experience and mathematical instruction children have in each of their languages.[7] For example, research on multilingual children's counting skills shows that as children first learn to recite numbers, they may show different counting levels in their various languages (for example, a child may recite numbers to 10 in English but only to six in Spanish). However, this research also finds that once children develop an understanding of cardinality, they can represent this knowledge using any of their languages.[8] This suggests that multilingual children may show differences in how skillfully they use mathematical language (for example, number words) but can represent new mathematical knowledge in all languages they know. Through collaboration with families, teachers can check for children's understanding in their home languages.

Developmental differences and varying needs can also impact the pace at which children learn mathematical ideas and skills and may require teachers to provide additional supports. For example, deafness can impact the way in which children learn to count, measure, pattern, and problem solve.[9] Research suggests that children who are deaf or hard of hearing often learn to recite numbers later than same-age peers in part because they may have less experience with mathematical language and thus have fewer opportunities to learn to count.[10] All children can demonstrate their mathematical knowledge when provided the appropriate supports and adaptations, including environment adaptations (for example, changes in seating or minimization of distractions), material adaptations (for example, technologies or adaptive equipment), and instructional adaptations (for example, different communication modalities and modeling). For children with disabilities, teachers should reference the Individualized Education Program (IEP) and regularly communicate with a child's IEP team to assist in making accommodations.

It is also important to consider that children may communicate their understanding in various ways through verbal language, nonverbal gestures, sign language, and other forms of communication, such as a picture exchange communication system or an electronic assistive technology communication device. Collaborating with a team consisting of teachers, counselors, occupational therapists, and psychologists, teachers should provide children with a variety of means to fully engage in mathematical tasks and encourage children to express their thinking in various verbal and nonverbal ways.

How Teachers Can Support Children's Early Mathematics Learning

"Teacher" refers to an adult (for example, lead teacher, assistant teacher, child care provider) with responsibility for the education and care of children in an early education program, including a California State Preschool Program, a Transitional Kindergarten program, a Head Start program, other center-based programs, and family child care homes.

Early childhood teachers play a pivotal role in helping young children develop their understanding of mathematical concepts and skills. Preschool-aged children benefit from regular, systematic, planned mathematics activities with specific learning goals, which can be part of an evidence-based curriculum with a scope and sequence. They benefit from a wide range of opportunities to solve problems, explore open-ended materials and practice mathematical language during play, routines, and everyday interactions. Research shows that children who experience a rich mathematical environment at home and in an early education program perform better in mathematics in elementary school.[11]

Exploring During Routines, Everyday Interactions, and Play

Mathematics is part of many aspects of children's lives. Children can learn about mathematical concepts through daily routines (for example, mealtimes, grocery shopping, setting the table), interactions with peers and adults, and play. Play provides children with meaningful and joyful opportunities to engage in mathematical thinking and problem-solving (for example, deciding how to evenly share four shells between two peers). As they play, children can develop deeper conceptual understanding of mathematical ideas by making connections between seemingly abstract concepts and real-world experiences (for example, through play children begin to understand that sharing is a real-world example of division).[12] Teachers can facilitate this learning by inviting children to solve problems they encounter in their play and asking open-ended questions to expand and deepen their learning (for example, "How do you know that you both have the same number of shells?").

Routines, everyday interactions, and play also provide teachers with opportunities to introduce children to related mathematical language. Modeling mathematically rich communication in the home or classroom languages will help children understand how mathematical vocabulary relates to the ideas they are exploring. For example, while children are building towers with blocks, teachers can model the use of vocabulary such as "tall," "taller," and "tallest." Furthermore, children benefit from being encouraged to express their understanding of mathematical ideas in multiple ways, using concrete objects, words, gestures, drawings, and symbols.

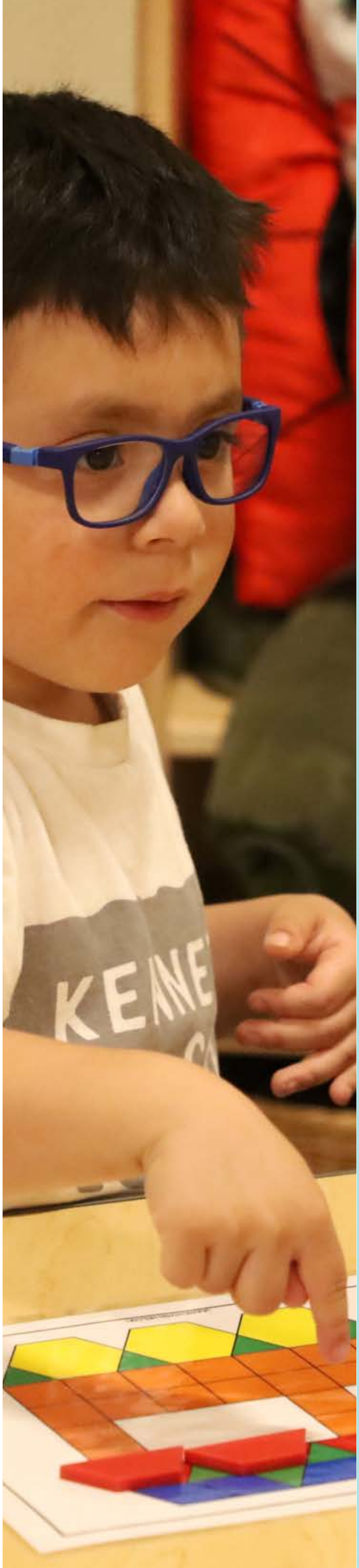
Setting Up Engaging Environments with Open-Ended Materials

Learning environments with varied, open-ended materials allow children to explore mathematical ideas and develop important mathematical skills in the context of teacher-planned mathematics activities as part of an evidence-based math curriculum, everyday interactions, and play. Open-ended materials that vary in attributes such as size, shape, or color allow children to practice sorting, creating patterns, measuring, and composing shapes. The open-ended nature of these materials also supports children of diverse backgrounds, skill levels, and interests to take part in mathematical learning. When available, technology can be used to represent mathematical problems and ideas in new ways, such as asking children to solve puzzles or play shape-matching games on a computer. Thus, providing children with a variety of objects and materials that promote mathematical exploration is essential for their mathematical learning.

Opportunities for Meaningful Investigations and Home Connections

Young children should be encouraged to follow their interests and ask questions about mathematical ideas they encounter through daily routines and interactions. Children can explore mathematical ideas in deeper ways when those ideas are presented in contexts that are meaningful, relevant, and individualized. One way teachers can nurture children's interests is by integrating children's home languages, cultures, and racial and ethnic backgrounds into mathematical learning experiences. For example, teachers may decide to introduce ways for children to engage in mathematical skills when preparing cultural foods (for example, measuring when preparing a cultural dish).

Given the important role that families play in children's mathematical learning,[13] teachers can learn from families how mathematics is already part of the families' daily activities with their young children and build on those experiences in the early childhood classroom. Teachers also can engage families and communities in children's mathematical learning and offer ideas about how to incorporate mathematics into their daily lives and home routines (for example, counting while grocery shopping, measuring while cooking, sorting while doing laundry).



Endnotes

1 M. R. Dillon, V. Izard, and E. S. Spelke, "Infants' Sensitivity to Shape Changes in 2D Visual Forms," *Infancy* 25, no. 5 (September 2020): 618–639; F. Xu and E. S. Spelke, "Large Number Discrimination in 6-Month-Old Infants," *Cognition* 74 (January 2000): B1–B11.

2 G. J. Duncan et al., "School Readiness and Later Achievement." *Developmental Psychology* 43, no. 6 (November 2007): 1428; A. Claessens and M. Engel, "How Important Is Where You Start? Early Mathematics Knowledge and Later School Success," Teachers College Record 115, no. 6 (June 2013): 1–29; T. Nguyen et al., "Which Preschool Mathematics Competencies Are Most Predictive of Fifth Grade Achievement?" *Early Childhood Research Quarterly* 36 (3rd quarter 2016): 550–560.

3 Nguyen et al., "Which Preschool Mathematics Competencies," 550–560; N. C. Jordan et al., "Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes," *Developmental Psychology* 45, no. 3 (June 2009): 850.

4 The Mathematics domain also aligns with *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve*. Relevant Content Connections (CCs) in Transitional Kindergarten (with Big Ideas in parentheses) include Reasoning with Data (Measure and Order, Look for Patterns), Exploring Quantities (Measure and Order, Count to 10), Taking Wholes Apart, Putting Parts Together (Create Patterns, Look for Patterns, See and Use Shapes), and Discovering Shape and Space (See and Use Shapes, Make and Measure Shapes, Shapes in Space).

5 Navajo People, Navajo Rugs, 2023, https://navajopeople.org/navajo-rugs.htm.

6 O. Cankaya, J. LeFevre, and K. Dunbar, "The Role of Number Naming Systems and Numeracy Experiences in Children's Rote Counting: Evidence from Turkish and Canadian Children," *Learning and Individual Differences* 32 (May 2014): 238–245; W. Mark and A. Dowker, "Linguistic Influence on Mathematical Development Is Specific Rather Than Pervasive: Revisiting the Chinese Number Advantage in Chinese and English Children," *Frontiers in Psychology* 6 (February 2015): 203.

7 B. W. Sarnecka, J. Negen, and M. C. Goldman, "Early Number Knowledge in Dual-Language Learners from Low-SES Households," in *Language and Culture in Mathematical Cognition*, eds. D. B. Berch, D. C. Geary, and K. M. Koepke. (San Diego, CA: Elsevier Academic Press, 2018), 197–227.

8 K. Wagner et al., "Why Is Number Word Learning Hard? Evidence from Bilingual Learners," *Cognitive Psychology* 83 (December 2015): 1–21.

9 C. M. Pagliaro and K. L. Kritzer, "The Math Gap: A Description of the Mathematics Performance of Preschool-Aged Deaf/Hard-of-Hearing Children," *The Journal of Deaf Studies and Deaf Education* 18, no. 2 (January 2013): 139–160.

10 S. Santos and S. Cordes, "Math Abilities in Deaf and Hard of Hearing Children: The Role of Language in Developing Number Concepts," *Psychological Review* 129, no. 1 (August 2022): 199–211.

11 E. C. Melhuish et al., "Effects of the Home Learning Environment and Preschool Center Experience upon Literacy and Numeracy Development in Early Primary School." *Journal of Social Issues* 64, no.1 (March 2008): 95–114.; S. Lehrl, K. Kluczniok, and H. G. Rossbach, "Longer-Term Associations of Preschool Education: The Predictive Role of Preschool Quality for the Development of Mathematical Skills Through Elementary School," *Early Childhood Research Quarterly* 36 (2nd quarter 2016): 475–488.

12 J. M. Zosh et al., *Learning Through Play: A Review of the Evidence* (Billund, Denmark: LEGO Foundation, 2017), 1–40.

13 E. C. Melhuish et al., "Effects of the Home Learning Environment and Preschool Center Experience upon Literacy and Numeracy Development in Early Primary School." *Journal of Social Issues* 64, no.1 (March 2008): 95–114.; C. Galindo, S. Sonnenschein, and A. Montoya-Ávila, "Latina Mothers' Engagement in Children's Math Learning in the Early School Years: Conceptions of Math and Socialization Practices," *Early Childhood Research Quarterly* 47 (2nd quarter 2019): 271–283.

Preschool/Transitional Kindergarten Learning Foundations in the Domain of Mathematics

Mathematical Practices

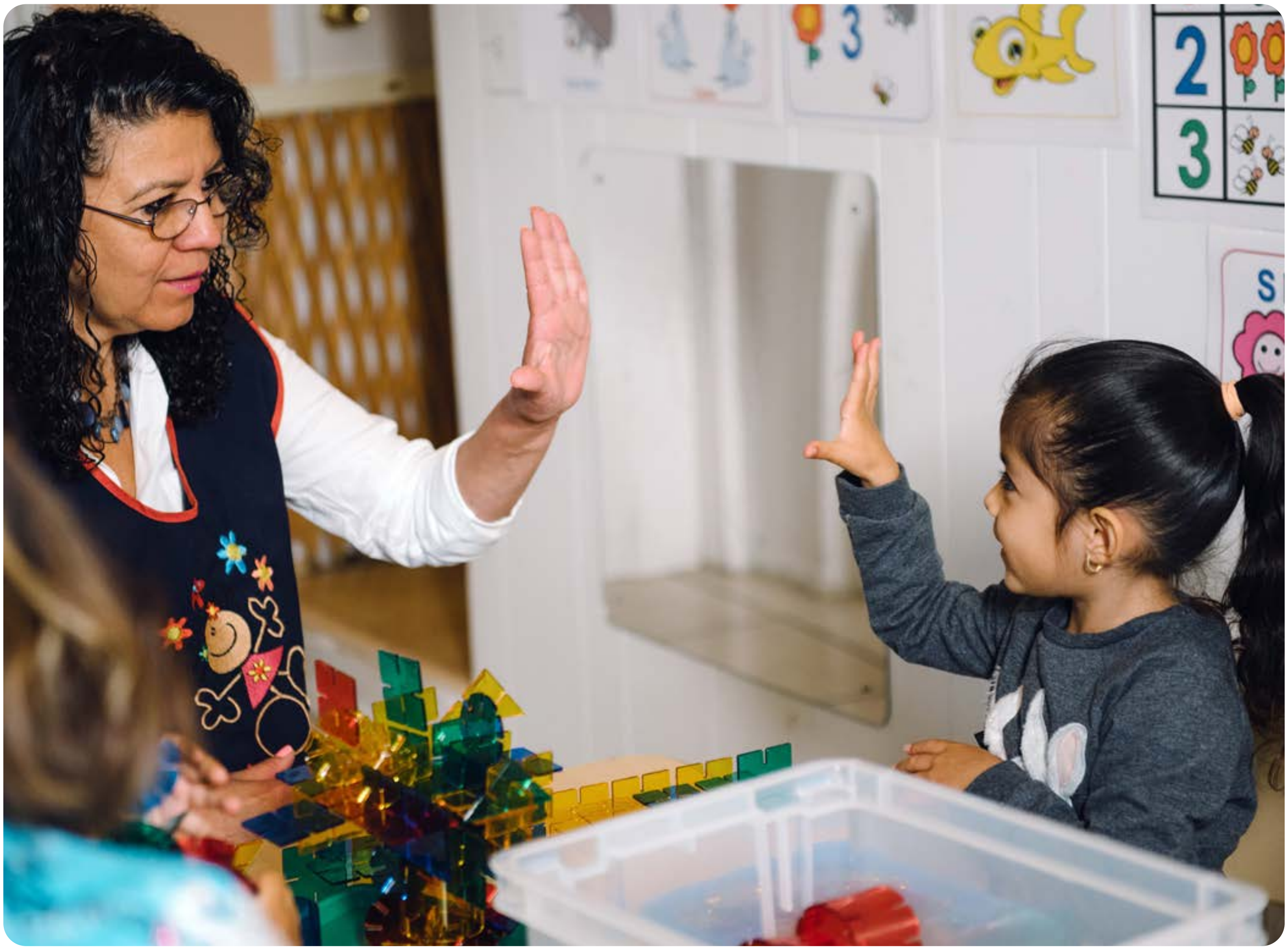
Mathematical practices describe the types of behaviors and dispositions that allow children to develop knowledge and skills in mathematics. These Mathematical Practices are identical to the Common Core Standards for Mathematical Practice with added descriptions that show how these practices apply to young children.

Teachers can support children in developing the skills and dispositions described in the mathematical practices by providing a rich learning environment that allows children to apply their reasoning and problem-solving skills in meaningful math explorations as part of everyday activities and through intentionally planned learning experiences provided in an evidence-based math curriculum. Teachers can also model the use of appropriate mathematical tools (for example, rulers), models (for example, a map), or strategies (for example, using your fingers for addition) to illustrate various ways a mathematical problem can be solved. Additionally, by asking open-ended questions, teachers can guide children in noticing patterns or rules, encourage children to explain their reasoning, and help children make connections between abstract mathematical ideas (for example, "taking away") and everyday situations (for example, finding out how many cars are left after giving some to a peer).

Mathematical Practices apply to children 3 to 5 ½ years of age (for preschool and Transitional Kindergarten)

1. **Make sense of problems and persevere in solving them.**
   Apply mathematical thinking to solve problems in everyday activities and interactions. Persist and try a variety of ways to solve a problem.
2. **Reason abstractly and quantitatively.**
   Reason about abstract mathematical ideas (for example, quantity, equality) using concrete objects (for example, "I have two cars and you have two. We have four cars."). Over time, relate to mathematical problems in more abstract ways (for example, two plus two equals four).
3. **Construct viable arguments and critique the reasoning of others.**
   Construct arguments about solutions, explain reasoning, and think critically about the solutions of others.
4. **Model with mathematics.**
   Use models (for example, drawings, constructions, modeling with their own bodies) to illustrate thinking and solve mathematical problems.
5. **Use appropriate tools strategically.**
   Use a variety of tools to solve mathematical problems (for example, a piece of string, a ruler, or a scale for measuring or ordering objects).
6. **Attend to precision.**
   Apply mathematical skills with increased precision (for example, counting, comparing, adding, subtracting).
7. **Look for and make use of structure.**
   Recognize structures and rules in mathematics (for example, all triangles have three sides).
8. **Look for and express regularity in repeated reasoning.**
   Notice patterns and regularity in mathematics (for example, a whole number plus one is the next number in the number list).

Children communicate their mathematical knowledge and skills in a variety of ways, both verbally and nonverbally. Their communication may include verbal ways of communicating in their home languages, the language of instruction, or a combination of languages, or through the use of augmentative and alternative communication devices. It may also include nonverbal ways of communicating such as drawing and modeling with different materials or expressing through movement, actions, or role-play.



Strand: 1.0 — Counting and Cardinality

Sub-Strand — Counting Principles

Foundation 1.1 **Reciting Numbers**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Recite numbers in order from one to ten or higher with no more than a few errors.	Recite numbers in order from one to thirty with no more than a few errors. Count forward from a number other than one.

Languages vary in their number naming system. Some languages, such as Chinese or Japanese, have a number system in which number words map directly onto a base-ten counting structure (for example, the word for 11 in Chinese is equivalent to "ten-one"), so the meaning of number words past ten is more obvious and may be easier to learn initially. In languages such as English and Spanish, some of the number words do not map clearly onto a base-ten structure (for example, the word for 11 in English is "eleven"). Given these differences in number naming systems, children may have an easier time learning to recite numbers in some languages than in others.

Early Examples

■ While playing, a child recites, "One, two, three, four, five, seven, nine, ten." The teacher says, "Let's try again and count to 10 together. One …" The child joins in and they count to 10 together.

● When the teacher asks a child, "What comes after five?" the child counts, "One, two, three, four, five, six! It's six!"

A Deaf child uses American Sign Language to recite the numbers one to 10.

Later Examples

■ While playing, a child recites one to 20 but forgets to say 14. The teacher comments, "Oops, I think you forgot a number. Let's start counting again from 10. Ten, eleven…" The child joins in and counts together with the teacher.

● When the teacher asks a child, "What comes after 11?" the child communicates, "12."

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 1.1 **Reciting Numbers**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

The teacher tells a child, “Let’s count to 10 while we wash our hands. One …” A child chimes in and counts to 10 with the teacher while washing their hands.

A child recites one to 12 while singing in Spanish.

Later Examples (continued)

A child starts counting as high as they can and after reaching 29, they count, “Twenty-ten.” Then the teacher provides guidance and says, “After 29 comes 30.” The child continues counting and stops at the next decade (40) with help from the teacher.

When waiting in line to go outside, a teacher asks if everyone can count to 20. The teacher encourages a child with a disability to use their communication device to count. The child uses their communication device to recite the numbers one to 20.

Foundation 1.2 **One-to-One Correspondence**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Count five objects or more using one-to-one correspondence (one object for each number word).	Count ten objects or more using one-to-one correspondence (one object for each number word).

Early Examples

■ While counting five blocks, a child points to the first block and says, "One," then points to the next block and says, "Two." The teacher points to each object along with the child as they count five blocks.

● When preparing for snack time, a teacher asks a child, "Would you please count the number of Asian pears on this tray?" The child counts the same Asian pear twice. The teacher encourages the child to try again and points together with them. The child correctly counts to six.

Later Examples

■ While counting 12 blocks, a child moves each counted block to a new pile to keep track of which blocks have already been counted. As the child places the 12th block on the new pile, the teacher points out, "I see you put the blocks you have already counted over here."

● When preparing for snack time, a teacher asks a child, "Would you please count the number of Asian pears on this tray? I want to make sure we have enough for all our friends." The child points to and counts each Asian pear and says, "We have 12."

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Foundation 1.2 **One-to-One Correspondence**

Early 3 to 4½ Years	Later 4 to 5½ Years
Early Examples (continued)	**Later Examples (continued)**
A child points to each wooden puzzle piece and counts in their home language, "One, two, three, four, five."	A child who is hard of hearing points to a flower in the garden and using speech and sign language communicates, "One," then points to another flower and signs, "Two." The child counts up to 12 different flowers. The teacher follows along and repeats the signs to encourage the child to keep going.
	A child counts the number of crayons in a box but counts the same crayon twice and misses some others. The teacher takes the crayons out of the box, lines them up in a row, and asks the child to count again by pointing to each crayon. This time the child successfully counts 11 crayons.

Foundation 1.3 **Cardinality**

Early **3 to 4 ½ Years**	Later **4 to 5 ½ Years**
Answer the question "How many?" by counting. May repeat the last number word in the number list after counting but is still developing an understanding that the number name of the last object counted represents the total number of objects in the group.	Consistently demonstrate understanding when counting that the number name of the last object counted represents the total number of objects in the group.

In the early foundation, children may answer "how many" by repeating the last number word in the number list after counting (for example, say "six" after counting "one, two, three, four, five, six"), but may still be developing an understanding that the number name of the last object counted represents the total number of objects in the group (for example, understanding that "six" represents that there are six objects in the group). However, in the later foundation, children demonstrate a deeper understanding of the quantity in the group (for example, communicating "six" and demonstrating understanding that it represents the total number of objects in the group).

Early Examples

■ When the teacher asks a child during an activity with five footprints, "How many animal footprints do you see?" the child counts, "One, two, three, four, five."

● While playing with teddy bears, a peer asks, "How many teddy bears do you have?" The child counts in Spanish, "One, two, three, four."

Later Examples

■ When the teacher asks a child during an activity with five footprints, "How many animal footprints do you see?" a child counts the footprints and communicates to the teacher, "I see five."

● While a child is playing with teddy bears, a peer asks, "How many teddy bears do you have?" The child counts in Spanish, "One, two, three … [and continues] … 14, 15," then says to the peer, "I have 15. How many do you have?"

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 1.3 **Cardinality**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

A child looks in the mirror in the playhouse and points to the braids in their hair. The teacher asks, “I see you have braids in your hair today, Amari. How many braids do you have?” The child counts, “One, two, three,” then holds up three fingers.

During lunchtime, a teacher asks, “How many plantain chips do you have?” and the child counts aloud the number of chips: “One, two, three, four, five, six—six.” When a peer then asks, “How many do you have?” the child starts counting the number of chips on their plate again.

Later Examples (continued)

On a walk outside, the teacher asks, “Does everyone have five rocks?” A child counts their five rocks and shares with the teacher, “I have five.”

When reading the book *Señorita Mariposa* with the teacher, a Deaf child counts the number of butterflies on the page using sign language and communicates, “Seven butterflies.”

A child counts five lychees* and recognizes there is one lychee for each of the five children around the table.

*A lychee is a small round fruit with a red husk and white flesh that grows in tropical regions.

Supporting Counting Skills and Understanding of Cardinality

Counting is one of the most fundamental skills in mathematics and involves many important concepts and skills. First, for successful counting, children need to be able to recite numbers. They also need to assign only one number word for each object (one-to-one correspondence) and count each object only once. Lastly, they need to understand that the number name of the last object counted represents the total number of objects in the group (cardinality). In addition to using an evidence-based mathematics curriculum with playful activities that follow a specific scope and sequence, teachers can support children in developing an understanding of counting during everyday interactions and routines by:

- Encouraging children to count during everyday routines, whether in English or in their home language with peers, family members, or community members who speak their home language. For example, teachers can encourage children to count to 10 while washing their hands.
- Modeling counting sets of objects, pointing to each object in a set and repeating the total number of objects. For example, a teacher can point to each rubber duck in a set of four while saying, “One, two, three, four,” then repeating, “There are four rubber ducks in the water.”
- Starting with counting small sets of objects. Once children have developed an understanding of cardinality and are comfortable applying one-to-one correspondence while counting small sets, asking them to count larger sets (for example, 12, 20, 30, and beyond) will help them develop an understanding of the base-ten number system.
- Encouraging children to use counting to answer “How many” questions and solve problems during everyday routines, learning experiences, and interactions with adults and peers. For example, teachers can ask children to count the number of apple slices on their plate or, while building, to find out which tower has more blocks.

Sub-Strand — Recognizing Quantities

Foundation 1.4 **Subitize**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Identify without counting the number of objects in a small collection (for example, one to four objects).	Identify without counting the number of objects in a collection of one to five objects.

Early Examples

■ A child looks briefly at a picture with three cats and immediately signs the quantity three.

● A child looks at the two crayons in their hand and shares with a peer, “I have two,” while holding out the two crayons.

A child turns toward their peer at snack time and expresses in Tagalog, “Ying has two strawberries.”

A child with low vision tosses a pair of tactile dice, touches the raised bumps on each die, and communicates, “One and three.”

Later Examples

■ A child looks briefly at a picture of five cats and immediately signs the quantity five.

● A child looks at the four crayons in their hand and shares with a peer, “I have four,” while holding out the four crayons.

During story time, a teacher briefly shows a picture of five ladybugs, then hides it behind their back and asks the class, “How many ladybugs did you see?” One child responds, “Five!”

While cleaning up after lunchtime, a child points to the four chopsticks on the table and comments to a peer in Mandarin, “Four chopsticks.”

A child points to the five cars in the toy car garage and gestures that there are five cars.

■ ● Matching icons indicate alignment of examples across age-ranges

Sub-Strand — Numeral Recognition

Foundation 1.5 **Numeral Recognition**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Recognize and name a few written numerals under 10.	Recognize and name all written numerals through 10.

Early Examples

■ While playing a card game, a child identifies the numeral three on the card and communicates, "Three." Then the child points to a card that displays the numeral seven and says, "One."

● When walking outside, a child points to the numeral eight in the house number "827" and says in Cantonese, "That's eight."

While reading a counting book, a Deaf child points to the numerals on the page and signs the correct number words. When the child gets to nine, they hesitate and sign, "I don't know." The teacher responds by showing them the sign for nine.

A child communicates, "That's a one," when playing with magnetic numerals. The teacher then asks the child to find the two magnet, but the child points to the three magnet. The teacher says "That's three. Remember, the two has a curve at the top like a three, but it has a straight line at the bottom. Can you find the two?" The child then correctly finds the two magnet.

Later Examples

■ While playing a card game, a child points to each numeral and names it, "Five, seven, two, one."

● When walking outside, a child points to each digit in the house number "827" and says in Cantonese, "That's eight, two, and seven."

While playing a computer matching game, a child incorrectly matches the numeral six to a picture of nine snails. The teacher encourages the child to try again by first counting the snails. After the child counts to nine, the teacher asks, "What does a nine look like?" When the child points to the numeral six, the teacher responds, "Six and nine look very similar, but remember nine has the circle at the top." The child then correctly chooses the numeral nine.

A child points to and names the numerals in a one-to-ten hopscotch painted on the ground as another child plays hopscotch.

■● Matching icons indicate alignment of examples across age-ranges

Sub-Strand — Number Relationships

Foundation 1.6 **Number Comparison**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Compare (with or without counting) two groups of objects that are clearly equal or different in size and communicate, "same" or "more."	Compare two groups of objects by counting and communicating, "more," "same," "less," or "fewer."

Early Examples

■ A child looks at the number of rocks they have and the number of rocks a peer has and communicates in their home language, "These are the same."

● While playing in the sandbox, a child looks at a peer's sand toys and comments in Mandarin, "You have more."

When looking over at a peer's plate, a child explains, "Jamal and I have the same," when referring to the number of carrots they each have at snack time. When the teacher asks, "How do you know you have the same amount?" the child lines up their carrots next to Jamal's carrots and explains, "Because look." The teacher responds by counting both rows and says, "I see, because you have one, two, three, four carrots and Jamal has one, two, three, four carrots too!"

Later Examples

■ A child counts the number of rocks they have and the number a peer has and communicates in their home language, "Eight and eight—you have the same as me."

● While playing in the sandbox, a child counts their own sand toys, then counts a peer's sand toys and comments in Mandarin, "You have more. I have three, and you have five."

When the teacher asks if everyone collected five leaves outside, a child looks over at a peer and comments, "They have less." The teacher then asks, "How do you know Arjun has less?" The child proceeds to count their own leaves and Arjun's leaves and describes, "Because I have five, but Arjun has three."

A child compares a group of seven shells with a group of nine shells by lining up the shells in each group side by side. The child points to the line with seven shells and shares, "This one has less."

■ ● Matching icons indicate alignment of examples across age-ranges

Strand: 2.0 — Operations and Algebraic Thinking

Sub-Strand — Number Operations

Foundation 2.1 **Principles of Addition and Subtraction**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Demonstrate understanding that adding or taking away one or more objects from a group will increase or decrease the number of objects in the group.	Demonstrate understanding that adding one or taking away one object changes the number in a small group of objects by exactly one.

Early Examples

■ While building with pine cones outside, a child grabs another pine cone from a peer and communicates, "Now I have more!"

● A child removes one stuffed animal from a collection of eight stuffed animals and gives it to a peer. The child comments, "Now I have less, and you have more."

While playing bakery, a teacher asks a child, "I would like two *bao** buns please." The child hands two *bao* buns to the teacher. The teacher asks, "Do you have any buns left?" The child responds, "All gone! I need to make more."

*Bao are steamed buns stuffed with savory and sometimes sweet fillings in the cuisine of Chinese communities.

Later Examples

■ While building with pine cones outside, a child counts their pine cones and says to a peer, "We have five." Then the child adds another pine cone to their structure and says, "Now we have six."

● A child removes one stuffed animal from a collection of eight stuffed animals and gives it to a peer. The child comments, "I have seven now." The child then counts the peer's stuffed animals and comments, "You have four now."

A child with autism using a communication device correctly predicts that if one more marble is added to a group of four marbles, there will be five.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.1 **Principles of Addition and Subtraction**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

A child gives their peer two dolls and describes in Spanish, “Now you have more dolls than me.”

Later Examples (continued)

While decorating a traditional Hmong hat with pom-poms to celebrate the Hmong New Year, the teacher asks everyone to add five pom-poms to the front of their hat. A child counts the pom-poms already on the hat, then grabs an additional pom-pom and describes, “I had four, but now I have five!”

Supporting Understanding of Addition and Subtraction

Children encounter addition and subtraction problems during everyday interactions, routines, and play with peers and adults. For example, children may combine their toy cars and a peer's cars and then find out how many cars they have altogether, or they may want to know how many crackers they have left after eating a few. In addition to using an evidence-based mathematics curriculum with playful activities that follow a specific scope and sequence, teachers can support children in developing an understanding of addition and subtraction during everyday interactions and routines by:

- Introducing children to simple addition and subtraction problems during daily routines. For example, teachers can ask children questions about change in quantity during playtime ("We have three boys and four girls on the bus. How many children are on the bus altogether?") or during mealtime ("I have five crackers. If I give you two crackers, how many crackers will I have left?").
- Using manipulatives, toys, or visual images when introducing children to simple addition and subtraction problems. For example, while reading a story to children, a teacher can point out that there are two ducks on the left page and three ducks on the right page and ask them how many ducks they see altogether. Modeling a problem with objects or visual images can help children visualize number **operations** such as adding to, combining, and taking away.
- Modeling the "count on" addition strategy. Younger children may use the "count all" strategy to add items in two sets. For example, they may add three red cars and two blue cars by counting them all ("one, two, three, four, five"). Teachers can encourage children to use the more advanced "count on" strategy by modeling using it, for example, by saying, "You have three red cars and two blue cars—three [pointing to the third red car], four, five [pointing consecutively to each of the two blue cars]."
- Using a variety of language to describe addition and subtraction problems. Teachers can start by using everyday language (for example, "altogether" and "take away"), then introduce formal mathematical language (for example, "add," "subtract," and "equal") and mathematical symbols (+ and –). This will help children make connections between the symbols of arithmetic (+ and –) and their everyday experiences in solving addition and subtraction problems.

Foundation 2.2 **Number Composition and Decomposition**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Demonstrate understanding that a set of objects is made up of smaller parts and that the whole set is bigger than its parts.	Decompose a set of objects into two small sets in more than one way (for example, decompose 5 into sets of 3 and 2, or 1 and 4). Combine two small sets to create a larger set (for example, 3 and 2 to make a set of 5).

Early Examples

■ While pretending to cook in the dramatic play area, a child explains to a peer, "You have two green apples, and I have only one red apple. Let's put all the apples together to make an apple pie."

● A child points to a collection of six puppets and communicates in their home language, "You get the bear puppets. I get the wolf puppets."

When reading the book *Baby Goes to Market*, the teacher asks, "What is happening to Mama's basket?" A child replies, "It is full!" The teacher asks, "Oh! What did the baby put in the basket to make it full?" The child replies, "Bananas and oranges."

Later Examples

■ While pretending to cook in the dramatic play area, a child explains to a peer, "We need five apples to make a pie, three red and two green. Actually, let's use four red and one green."

● A child points to a collection of six puppets and communicates in their home language, "Three bear puppets for me, and three wolf puppets for you." After playing for a while, the child says, "Now I want two bear puppets and one wolf puppet."

When acting out a story, a child removes three (of four) ducks from the flannel board, communicating, "Three left the pond, and only one stays."

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.2 **Number Composition and Decomposition**

Early 3 to 4½ Years	Later 4 to 5½ Years
Early Examples (continued)	**Later Examples (continued)**
During snack time, a child points to their animal crackers and expresses using a mix of English and Spanish, "*Tengo cuatro* animal crackers. Two elephants *y* two *jirafas*." (I have four animal crackers. Two elephants and two giraffes.)	A child indicates five by holding up three fingers on one hand and two fingers on the other. The teacher asks, "How else can you show five?" After some thinking, the child holds up one finger on one hand and four fingers on the other.
A child with a disability combines their magnetic tiles with a pile of a peer's magnetic tiles and uses their communication device to show the peer that the peer's pile has more magnetic tiles now.	

Promoting Number Composition and Decomposition Skills

Number composition is the understanding that two parts can be put together to form a whole (for example, 1 and 3 make 4), whereas number decomposition is the understanding that a whole can be broken into smaller parts (for example, 5 can be broken into 3 and 2). Through composition and decomposition of numbers, children learn how numbers can be represented in different ways (for example, 5 = 4 + 1 and 5 = 2 + 3). Number composition and decomposition reinforces children's understanding of addition and subtraction and lays the foundation for understanding **place value**. In addition to using an evidence-based mathematics curriculum with playful activities that follows a specific scope and sequence, teachers can support children in developing number composition and decomposition skills during everyday interactions and routines by:

- Providing a variety of materials and tools to help children work through composition and decomposition problems. This might include objects, manipulatives, and containers for splitting up sets of objects. For example, a teacher can provide children with two baskets and ask them to split five counting bears into the two baskets. Once they have tried one way, the teacher can ask them to show a different way of splitting the bears.
- Inviting children to practice composition and decomposition through a variety of activities. For example, a teacher can invite children to form a group of six using different toy animals (such as one rooster, two cats, and three frogs).
- Asking questions to encourage children to think about composition and decomposition and the structure of numbers, for example, "You have 12 toy cars in the parking lot you made, 10 in this row and two in the next row. What other ways might you arrange these cars? How might you arrange these cars in three rows? How many cars would be in each line? How many cars would there be altogether?"

Foundation 2.3 **Solving Addition and Subtraction Problems**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Solve addition and subtraction problems with a very small number of objects in the context of everyday situations.	Solve addition and subtraction problems with a larger number of objects (sums up to 10) in the context of everyday situations.

Early Examples

■ During lunch, a teacher asks a child to serve their peer one samosa* from a plate of four samosas. The child counts the samosas that remain and explains, "Now there are only three left."

● When reading a story, a child looks at the picture and counts the number of children going to school in a car and the number of children going to school in a bus. The teacher asks, "How many children are there altogether?" The child then counts, "One, two, three, four, five!"

*A samosa (or *singara*) is a filled savory pastry common in South Asian cuisine.

Later Examples

■ During lunch, a teacher asks a child to serve their peer two samosas from a plate of eight samosas. The child counts the samosas that remain and describes, "There were eight samosas. I gave you two, so now there are six."

● When reading a story, a child looks at a picture and counts three children going to school in a car and four children going to school in a bus. The teacher asks, "How many children are there altogether?" The child replies, "There are four in the bus, so … five, six, seven!"

While pretending to shop at a clothing store, a child tells a peer, "The coat is six dollars, and the pants are three dollars." The peer takes out their purse and counts out six dollars. The peer then adds three more dollars and expresses, "One, two, three, four, five, six, seven, eight, nine."

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.3 **Solving Addition and Subtraction Problems**

Early 3 to 4½ Years	Later 4 to 5½ Years

Early Examples (continued)

A child draws a canoe with three stick figures inside. When the teacher asks, "Three people in a canoe—do you think one more person would fit?" the child draws another stick figure and indicates the total number of people in the canoe by showing four fingers. The teacher replies, "Yes, now there are four people in the canoe."

A child recognizes that one ball together with another two makes a total of three balls and communicates "three" in their home language and then in English.

Later Examples (continued)

While playing at the water table with peers, a child who is blind communicates to their peer, "I have five boats." A peer asks if they can have two of the boats. The child replies, "If I give you two boats, I will only have three left."

While picking fruit from the classroom garden, a child adds two more blueberries to a group of seven and holds up nine fingers when the teacher asks how many blueberries they have altogether.

Foundation 2.4 **Sharing Objects (Division)**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Share a small number of objects (for example, four or six objects) equally between two recipients.	Share a slightly larger number of objects equally between two or more recipients (for example, nine objects among three recipients).

Early Examples

■ A child splits a row of four acorns down the middle, gives two acorns to a peer, and keeps the other two acorns.

● When working on an art project, a child hands one stamp to their teacher, keeps the other stamp, and communicates, “One for you. One for me.”

A teacher asks a child to share some stickers with a peer. The child takes turns giving one sticker to a peer and one sticker to themself until all the stickers are shared.

Later Examples

■ A child divides a row of eight acorns down the middle, gives half to a peer, and keeps the other half.

● When working on an art project, three children divide up nine stamps. They take turns taking one stamp each until none are left. Each child then counts their own pile of stamps to check that they all have the same number.

A child shares six stickers with a peer, keeping four and giving two to the peer. The peer says in Arabic, “That’s not fair.” The child gives the peer one more sticker and replies in Arabic, “Now we both have three stickers.”

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.4 **Sharing Objects (Division)**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**
	Later Examples (continued)
	When trying to divide several figurines between their peer and themself, a child keeps three figurines and gives three figurines to the peer, then signs, “I have three, and you have three.” The teacher signs, “What should happen with the last one?” The child replies that the last one will stay in the basket.
	A child says, “You get half, and I get half. We both get the same,” when dividing a row of eight quilt squares after a read-aloud of the book *Cassie’s Word Quilt*.

Sub-Strand — Classifying and Patterning

Foundation 2.5 Sorting and Classifying

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Notice similarities and differences in the attributes of objects. Sort and classify objects by one attribute into two or more groups.	Sort and classify objects by one or more attributes into two or more groups with accuracy and flexibility. When sorting by two attributes, a child may first sort by one attribute and then by the second attribute.

Early Examples

■ A child sorts toy dinosaurs by size and communicates, "I put all the big dinosaurs here and all the small ones there."

● While playing with toy animals, a child selects some giraffes for themselves and some elephants for a peer, leaving all the other animals unsorted.

During clean-up time in the dramatic play area, the teacher asks children to sort the bangles* by color. Two children put all the gold bangles in one basket and all the red bangles in another basket.

*Bangles are bracelets worn by women in Southeastern Asia and Africa. They are usually made from metal or glass.

Later Examples

■ A child sorts toy dinosaurs first by size (big and small) and then by color (brown, green, and orange) for each group of big and small dinosaurs.

● After learning about the different ways animals move, a child sorts a variety of toy animals into three groups: those that fly, those that walk, and those that swim.

A child arranges blocks in baskets according to shape: one basket with cylinders, another with square pyramids, and a third basket that includes both cubes and rectangular prisms. The teacher points to the third basket and says, "I think you have two different shapes in this basket. Can you sort them?" The child then sorts the cubes and rectangular prisms into separate baskets.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.5 **Sorting and Classifying**

Early 3 to 4½ Years	Later 4 to 5½ Years

Early Examples (continued)

A child with limited arm mobility sorts crayons by color and tells the teacher in which colored basket each crayon goes. The teacher points to a red crayon and asks, “Where does this one go?” The child replies, “It goes in the red one.”

While building a house outside with rocks, a child chooses only large flat rocks and lays them down for the foundation, then stacks smaller rocks to build a wall.

Later Examples (continued)

When preparing to make a fruit salad, a child says in Vietnamese, “I’m gonna put the oranges in this bowl. The grapes go in this one, and the bananas will go here on this plate.”

Sorting and classifying are important precursors to children’s ability to represent and analyze data (see foundations 3.4 and 3.5). When working with children on skills related to data, consider ways children can use their sorting and classifying skills to sort data into groups.

Foundation 2.6 **Recognizing, Duplicating, and Extending Patterns**

Early 3 to 4 ½ Years	**Later 4 to 5 ½ Years**
Notice and explore patterns in their environment and, with adult support, duplicate simple repeating patterns (for example, ABAB).	Explore, extend, and duplicate a variety of repeating patterns (for example, AABBAABB, ABCABC) with adult support. Describe the repeating part of a pattern (pattern unit).

Early Examples

■ During circle time, the teacher claps their hands together and then taps their knees in a clap-tap-clap-tap pattern. A child participates by clapping their hands and tapping their knees.

● A child with low vision feels a bracelet made by a peer out of differently sized beads and communicates, "I want to make the same one. Big, small, big, small."

During outdoor playtime, the teacher calls out patterns for the children to follow such as hop-clap-hop-clap. Children follow the directions, occasionally making mistakes.

Later Examples

■ During circle time, the teacher plays a loud-loud-soft-soft pattern by hitting a *djembe** drum and asks a child to try the same pattern. After watching the teacher a few times, the child copies the same pattern.

● A child with low vision feels a bracelet made by a peer out of differently sized beads that follows the pattern "big, big, small, big, big, small." The teacher asks if the child wants to finish it by adding another set of beads. The child feels the beads and describes, "I need to add two big and one small!"

*A *djembe* is a type of drum from West Africa.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.6 **Recognizing, Duplicating, and Extending Patterns**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

While beading a necklace, a child proudly shows their aunt their pattern and communicates, "Look, white, blue, white, blue."

Later Examples (continued)

While reading a book about Día de los Muertos,* a child whose home language is Spanish points to a picture of a *papel picado*** banner and says, "Look, the colors repeat, *rosa*-blue-green-*rosa*-blue-green." (*Rosa* is pink in Spanish.)

When reading a picture book about the different phases of the moon, the teacher asks a child, "What is happening to the moon?" The child answers, "It gets smaller and smaller and then bigger and bigger."

*Día de los Muertos (Day of the Dead) is a holiday honoring the dead celebrated in Mexico and communities of Mexican heritage.

***Papel picado* is a type of folk art originating in Mexico in which intricate designs are cut into colorful tissue paper.

Foundation 2.7 **Creating Patterns**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Create, with adult support, a simple repeating pattern (for example, ABAB).	Create a variety of repeating patterns (for example, AABBAABB, ABCABC) or recreate existing patterns using different objects.

Early Examples

■ A child uses different colored buttons to create a mosaic. The child starts with a blue button, adds a yellow button, and then adds another blue one to make blue-yellow-blue. The teacher asks if a yellow one comes next. The child places a yellow button next to the blue one and responds, "Blue-yellow-blue-yellow."

● A child makes up a dance to a song by creating a clap-clap rhythm. The teacher then suggests they add in a stomp-stomp. The child tries out a clap-clap-stomp-stomp pattern in their dance.

During snack time, the teacher asks a child whether they can make a pattern with their pretzel sticks and cheese slices. The teacher starts the pattern with a pretzel stick and asks the child to decide what comes next. The child then creates a pretzel-pretzel-cheese-pretzel-pretzel-cheese pattern.

Later Examples

■ A child uses different colored buttons to create a mosaic. The child starts with a small purple circle, then adds alternating concentric circles of green, black, and purple buttons.

● A child makes up a clapping game with a peer where they alternate clapping their hands and stomping their feet repeatedly to the tune of "Miss Mary Mack."

The teacher shows a child picture cards with a bus-bus-car-bus-bus-car pattern and asks the child to make the same pattern using colored blocks. The child then makes a green-green-blue-green-green-blue pattern.

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■ ● Matching icons indicate alignment of examples across age-ranges

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Foundation 2.7 **Creating Patterns**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

During a class project about caterpillars, a child observes a caterpillar closely and notices the color pattern on its back. The child then fills in a picture of a caterpillar by adding green and yellow stripes.

Later Examples (continued)

A child decides to decorate their carnival* mask with sequins in a pink-gold-blue-pink-gold-blue pattern.

A child creates a picture by copying the cloud-sun pattern in a drawing by the Elder.

*Carnival is a festival that takes place right before Lent in many Spanish- or Portuguese-speaking countries. Carnival is marked by colorful costumes, music, dancing, and an abundance of food.

Developing Patterning Skills

A pattern is a regularly repeated arrangement of things such as numbers, objects, events, and shapes. From a young age, children notice patterns around them. They can predict what comes next in a pattern and notice if someone breaks a pattern. Teachers can help children develop their abilities to identify, describe, replicate, extend, and create patterns. This type of thinking is essential for children's algebraic thinking in elementary school. In addition to using an evidence-based mathematics curriculum with playful activities that follow a specific scope and sequence, teachers can support children in developing patterning skills during everyday interactions and routines by:

- Encouraging children to notice patterns in their environment and describe these patterns. For example, teachers can organize a pattern hunt and ask children to point out any patterns they notice in their classroom or while on a walk.
- Playing games that allow children to practice extending or duplicating patterns, for example, playing a pattern clapping game, introducing body-movement patterns while dancing to a song, or making patterns with beads while stringing a necklace.
- Inviting children to make their own patterns and supporting them in describing what the repeating part of a pattern (pattern unit) is, for example, "I see you made a pattern with these blocks. What comes next?"
- Introducing children to a variety of types of patterns, in a variety of modalities. Children are often presented with simple repeating patterns (for example, ABAB, ABBAABBA, ABCABC) during patterning activities. To broaden children's knowledge of patterns, teachers can introduce them to growing patterns (for example, a row of blocks that gets longer and longer), patterns in numbers (for example, skip counting by twos), or musical patterns (for example, an ascending melody).

Strand: 3.0 — Measurement and Data

Sub-Strand — Comparing and Ordering Objects

Foundation 3.1 **Comparing Measurable Attributes of Objects**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Demonstrate awareness that objects can be compared by length, weight, or capacity by noticing differences in objects and communicating about their comparison.	Compare two objects by length, weight, or capacity (for example, putting objects side by side) and communicate about their comparison.

The language children use to compare properties such as length, weight, or capacity will differ based on their home languages. In English, many comparative words are formed by adding a suffix to a descriptive word. For example, "long" becomes "longer," and "heavy" becomes "heavier." In other languages, such as Spanish, comparisons are made by adding "more" or "less" to comparative words. For example, más grande translates to bigger (more big), and menos pesado translates to less heavy. Children learning a language that uses this grammar may use phrases like "more heavier" when speaking in English.

Early Examples

■ A child looks at a peer and expresses, "You are more bigger than me."

● A child fills up a container with water all the way to the top and says, "Full," then points to a container that is almost empty and comments, "Empty."

Later Examples

■ A child tries to determine whether they are taller than a peer by standing next to the peer. The child compares their height and expresses to the peer, "You are a little bigger than me."

● A child fills up a container with water all the way to the top, then pours the water into a different container to find out which one holds more.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 3.1 **Comparing Measurable Attributes of Objects**

Early
3 to 4½ Years

Later
4 to 5½ Years

Early Examples (continued)

A child is putting their doll to bed and covers the doll's hair to protect it. The hair covering* falls off. The child notes, "Too big" and searches the pretend play bin for a smaller hair covering, then says, "This one fits better."

A child builds a tower next to another child and communicates in Tagalog, "Look, mine is taller."

A teacher has drawn two lines on a whiteboard and asks a child, "Can you tell which one is longer?" A Deaf child signs, "Long" and points to a long line drawn on the whiteboard. Then the child signs, "Short" and points to a shorter line on the whiteboard.

*Different communities use hair coverings for different purposes, for example, to protect their hair or for religious reasons. They have different names depending on the community and use, such as scarf, bonnet, durag, turban, hijab, and khimar.

Later Examples (continued)

With the help of a teacher, a child uses a balance scale to find out which of two rocks is heavier. When the teacher asks, "Which rock is heavier?" the child points to the side of the scale that went down. The teacher then asks, "How do you know this one is heavier?" The child responds, "Because it went all the way down."

While learning about stringed instruments from around the world, a child points to a picture of an *erhu*** and says, "This one has really long strings."

A child cuts a paper strip that is the same length as their foot using adaptive scissors. Then they compare the paper strip to the length of their arm and communicate to the teacher in Spanish, "My arm is longer than my foot."

**The erhu is a Chinese two-stringed bowed musical instrument.

Many of the concepts and skills described in the Mathematics domain are important for children's learning and development in other domains, particularly in science. Children use mathematics skills such as sorting, classifying, patterning, measuring, and collecting data while engaging in scientific inquiry and exploration.

Foundation 3.2 **Ordering Objects**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Order a few objects (for example, three) by length or other attributes (for example, height, capacity).	Order a slightly larger number of objects (for example, four or five) by length or other attributes (for example, height, capacity).

Early Examples

■ A child orders three bowls by size in the dramatic play area: the biggest bowl for daddy bear, the medium bowl for mommy bear, and the smallest bowl for baby bear.

● While playing at the water table, two children and a teacher are filling three water bottles with water beads. Halfway through filling the water bottles, the teacher says, "Can you order these bottles from most full to least full?" The children work together to compare the bottles and order them.

Later Examples

■ A child orders five bowls by size in the dramatic play area: the biggest bowl for daddy bear, the second-biggest bowl for nana bear, the middle-sized bowl for mommy bear, the next-smallest bowl for brother bear, and the smallest bowl for baby bear.

● While playing at the water table, two children and a teacher are filling five water bottles with water beads. Halfway through filling the water bottles, the teacher says, "Can you order these bottles from most full to least full?" One child orders the bottles and explains which bottles have the largest and smallest number of beads.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 3.2 **Ordering Objects**

Early 3 to 4½ Years	Later 4 to 5½ Years

Early Examples (continued)

A child with Down syndrome gestures to the teacher and points to three pumpkins they have ordered from big to small. The teacher responds, "You have lined them up from big to small."

A child lines up three animal figures—a giraffe, a horse, and a rabbit—by size. The teacher then asks, "Which one is the biggest?" The child points to the giraffe at the start of the line and communicates, "Big." The child then points to the rabbit at the end and says, "Small."

While children are making their own panpipes,* a child orders three different length straws from smallest to biggest before taping them together.

*Panpipes (or pan flutes) are a type of wind instrument made of thin tubes of different sizes. They are played in the southern Andean region of South America (Peru and Bolivia).

Later Examples (continued)

A child arranges four dolls from smallest to largest during pretend play.

In the sandbox, a child lines up five buckets by size, from the bucket that holds the most sand to the one that holds the least. The teacher notices and asks the child, "Why did you order the buckets like this?" The child explains that they arranged the buckets from small to big.

A child orders their five peers from shortest to tallest by comparing them to each other. They say to their tallest peer, "You are the tallest, so you go here."

Foundation 3.3 **Measuring Length**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
(No foundation)	Measure length using concrete objects laid end to end, sometimes needing adult support. Note: Children may not yet understand that units need to be of equal length.

Later Examples

A child uses paper clips laid end to end to measure the length of their hand and the hand of the teacher. Then they communicate, "Mine is four, and yours is seven."

A child measures the length of a rug by laying differently sized blocks end to end. When the teacher explains that the blocks all need to be the same, the child swaps out some of the blocks so they are all the same size.

A child measures their foot using a ruler and communicates, "Mine goes to six." They then measure the teacher's foot and explain, "Yours is nine."

A child measures the distance between two cedar trees in the play yard by counting the number of steps as they walk from one tree to the other.

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Foundation 3.3 **Measuring Length**

Early 3 to 4½ Years	Later 4 to 5½ Years
	Later Examples (continued) A teacher asks a child to help them measure the length of the bookshelf so they can order a second one for the library. With guidance from the teacher, the child puts the measuring tape at one end of the shelf and then pulls the tape across the length of the shelf. The teacher points to the number on the measuring tape and says, “That’s a two and a six, which means this is 26 inches long.”

Supporting Ordering and Measurement Skills

Young children develop an intuitive concept of measurement as they make comparisons between two objects to determine which one is longer, taller, or heavier. They also learn to identify slight variations in size and can order several objects from smallest to largest. Comparing and ordering skills are fundamental to children's understanding of more precise measurement concepts (for example, using constant units that are *laid end to end* to measure length). In addition to using an evidence-based mathematics curriculum with playful activities that follow a specific scope and sequence, teachers can support children in developing ordering and measurement skills during everyday interactions and routines by:

- Facilitating learning experiences that require children to compare, order, estimate, or measure to solve a problem. During these activities, teachers can invite children to reason and explain their thinking, for example, by saying, "I wonder if this piece of paper is big enough to cover the entire table. What can we do to check if it is the right length?"
- Modeling and encouraging children to use comparative and measurement vocabulary such as short, long, tallest, heavier, light, hot, slower, and fastest.
- Inviting children to make measurement estimates and then checking their answers using measurement tools. For example, when exploring rocks, teachers can ask, "Which rock do you think is heavier?" After children have made an estimate, teachers can guide children to check their estimate by using an appropriate tool such as a balance scale.
- Modeling the use of standardized measurement tools such as tape measures, rulers, stop watches, and scales. Teachers can demonstrate the importance of placing the tool at the origin, for example, when measuring the length of the table, showing how to line up the ruler starting at zero.

Sub-Strand — Data

Foundation 3.4 **Representing Data**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Use objects, tally marks, or pictographs to represent data in two groups, with adult support.	Use objects, tally marks, or pictographs to represent data in two or more groups. Demonstrate understanding that each object, tally mark, or picture represents one data point.

Early Examples

■ The teacher asks the children to add a block under the picture of their favorite fruit (mango or orange). A child communicates, "I like mango." The teacher responds, "So where should you put your block?" The child then adds a block under the picture of the mango.

● During a play dough–making activity, children are asked to vote for the color they want to make the play dough. Each child is asked to draw a line under the pink or the orange circle on the whiteboard. A child takes a marker with an adaptive grip and draws a line under the pink circle.

Later Examples

■ The teacher asks the children to add a block under the picture of their favorite fruit (mango, durian,* or orange). A child stacks their block on top of the tower next to the picture of a durian. A peer then adds a block to the tower next to the picture of a mango. The child points to each of the blocks and communicates in their home language, "This is mine and that is yours."

● During a play dough–making activity, children are asked to vote for the color they want to make the play dough. Each child is asked to mark an "X" under the pink, orange, or blue circle on the whiteboard. A child walks up to the chart, draws an "X" under the blue circle, and communicates, "This is my vote."

*Durian is a fruit commonly eaten in Southeast Asian cultures.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 3.4 **Representing Data**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

To see how many children in the classroom are wearing stripes, the teacher asks children to go to the dramatic play area if they are wearing stripes or the water table area if they are not. A child with a physical disability says to the teacher, “My shirt has stripes.” The teacher asks, “So which area should you go to?” The child points and says, “Over there” and heads to the dramatic play area in their wheelchair.

Later Examples (continued)

While playing outside, each child has a piece of paper with pictures of a leaf, an acorn, and a pine cone. When a child finds a leaf under the slide, they add a sticker under the picture of the leaf. Then they find an acorn next to the bench and add a sticker under the picture of an acorn.

Foundation 3.5 **Interpreting Data**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Notice, with adult support, differences in the data of two groups and describe which one has more or less.	Describe and compare, with adult support, the number of data points in two or more groups. Determine which group has more or less.

Early Examples

■ After all children have voted for their favorite fruit by adding a block under its picture, the teacher asks a child which side has more blocks. The child points to the side with the mango picture and comments, "This one has more." The teacher then asks, "Which fruit do the most children like?" and the child answers, "Mango."

● After all children have voted to make either pink or orange play dough, the teacher asks a child in their home language, "Which color got more votes?" The child points to the side of the whiteboard that has a pink circle. The teacher then asks, "Which color has fewer votes?" The child then points to the orange circle.

Later Examples

■ Children voted for their favorite fruit (mango, durian, or orange) by stacking same-sized blocks under their favorite fruit. The teacher asks what the class's favorite fruit is. A child brings the three block towers closer to one another, compares their heights, and explains while pointing to each tower, "This one is the tallest, this one is medium, and this one is smallest." The teacher responds by asking, "So which fruit is the favorite?" and the child answers, "Mango."

● After all children in the classroom have voted to make either pink, orange, or blue play dough, the teacher asks a child which color got more votes. The child begins counting the number of Xs under the pink, orange, and blue circles. The child counts eight votes for pink, two votes for orange, and four votes for blue and communicates that pink got more votes.

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■ ● Matching icons indicate alignment of examples across age-ranges

(continued)

Foundation 3.5 **Interpreting Data**

Early 3 to 4½ Years	Later 4 to 5½ Years

Early Examples (continued)

The teacher has asked children to gather in two different areas of the classroom based on whether they are wearing stripes or not. Children wearing stripes have gathered in the dramatic play area, and children who are not wearing stripes have gathered in the water table area. A child looks at both groups and comments that there are more peers in the dramatic play area. The teacher responds, "You are right. Most of our friends are wearing stripes today."

Later Examples (continued)

After spending a few minutes gathering data on the types of items (for example, leaves, acorns, and pine cones) they could find in the playground, a child counts the number of pictographs (stickers) under each category. The child counts four leaves, two acorns, and one pine cone. The child communicates to the teacher that they saw mostly leaves.

Strand: 4.0 — Geometry and Spatial Thinking

Sub-Strand — Shapes

Foundation 4.1 **Identifying Two-Dimensional Shapes**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Identify familiar two-dimensional shapes such as circle, square, triangle, and rectangle. Note: May not identify a nontypical version of a shape (for example, a square turned so that the point is down—a diamond).	Identify, describe, and construct different shapes including variations of circle, square, triangle, rectangle, and other shapes. Use informal language to describe defining properties of a shape (for example, sides, corners, round).

Early Examples

■ A child sorts shape manipulatives of different sizes into different shape groups. They point to the group of equilateral triangles and communicate, "Here are the triangles" and ignore the nontypical triangles (isosceles or right-angled triangles).

● A child uses play dough to construct a rectangle and shares, "Look! A square." The teacher responds, "Wow! You noticed this shape has four sides like a square, but it actually is a rectangle. It has two long sides and two short sides. See?" The teacher then runs their finger along the sides of the rectangle to emphasize their lengths.

Later Examples

■ A child sorts manipulatives of different sizes and orientations by shape and says, "This is a triangle because it has three sides." The teacher responds, "What if we turn it like this?" The child answers, "You try to trick me! It's still a triangle!"

● A child uses play dough to construct rectangles and squares of different sizes. The child communicates to the teacher, "Look! I made a square! All the sides are the same," while pointing to each of the sides.

While playing the I Spy the Shape game, a child says in Spanish, "I see a circle—the clock." Later, the child says, "I see a rectangle—the table."

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 4.1 **Identifying Two-Dimensional Shapes**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

During lunchtime, a child points to a roti* and expresses in their home language, "This is a circle."

While playing shape bingo, a child indicates the correct shape.

A child shows the dream catcher they made to their Elder and says, "The outside is a circle, but in the middle I made a triangle."

*Roti (or chapati) is a type of round unleavened flatbread common throughout Southeast Asia.

Later Examples (continued)

A child with a language impairment draws a rocket by tracing around square and triangular blocks. The teacher notices the picture and says, "I see you drew a rocket." The child nods their head "yes" and points to the triangle at the top of the rocket. "I see you added a triangle at the top to make the rocket pointy." The child then raises their arms above their head to make a triangle shape with their body and pretends to be the rocket.

While playing Guess the Shape, a child describes a shape to a peer by explaining that it has four sides that are all the same. One peer guesses, "Square."

Foundation 4.2 **Identifying Three-Dimensional Shapes**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Occasionally identify a few familiar three-dimensional shapes using informal names (for example, saying "ball" when referring to a sphere).	Identify a few familiar three-dimensional shapes such as sphere, cube, and cylinder. Note: Sometimes still use informal names (for example, ball, square box, tube).

Early Examples

■ While reading a book about foods from around the world, a teacher asks a child to point to the foods that have a cylinder shape. The child points to a Japanese sushi roll.

● A teacher invites a small group of children to sort blocks by shape. A child grabs a pyramid and says in their home language, "This one looks like a roof." The teacher asks, "Do you remember what we call this shape?" The child shakes their head, so the teacher responds, "This is called a pyramid."

A teacher invites a child to sort blocks of varying shapes into groups. The child then points to the group of cubes and communicates, "I put the square boxes here."

Later Examples

■ When looking at a book about the Lunar New Year, a child points to a picture with differently shaped lanterns and communicates, "This one is like a ball, and this one is a cylinder."

● A teacher invites a small group of children to sort blocks by shape. As one child is sorting, they explain to the teacher in their home language that all the pyramids have a square on the bottom and triangles on the sides.

While playing Guess the Shape, a child puts their hand in a bag and without looking begins to describe a shape by explaining that it has a flat circle on each side. A peer guesses that it is a tube. The child pulls the shape out of the bag, and the teacher communicates, "You are right. This is called a cylinder."

When playing outside, a child observes a roly-poly (pill bug) in the dirt and tells a peer in Cantonese, "Look, it turned round like a ball."

■ ● Matching icons indicate alignment of examples across age-ranges

Foundation 4.3 **Comparing Two-Dimensional Shapes**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Compare two-dimensional shapes of different sizes and orientations to determine whether they are the same shape.	Compare two-dimensional shapes of different sizes and orientations to determine whether they are the same shape. Identify similarities and differences in the properties (number of sides or vertices) of two different shapes.

Early Examples

■ When playing a matching game, a Deaf child uses American Sign Language to communicate that both shapes are squares.

● When playing Find the Shape, the teacher gives each child a shape and asks them to find that shape in the classroom. A child holding a circle cutout walks over to the clock and says in Arabic to a peer, "Circle!"

While playing in the block area, the teacher points to two rectangles and asks, "Are these the same shape?" A child turns the two rectangular blocks so the longest side is at the bottom, aligns them, and communicates, "Same!"

While playing Shape Hopscotch, a child jumps only on the triangle shapes to get from one side of the room to the other.

Later Examples

■ When playing a matching game, a Deaf child points to the three vertices of the first shape and signs, "This is a triangle because it is pointy at the top—see?" Then they point to the second shape and sign, "This one has four sides, so it is a rectangle."

● When playing Find the Shape, a child pulls all the triangles, big and small, from around the classroom to one side and explains in Arabic to a peer, "These are all triangles because they have three sides."

A child sorts pictures of various circles and rectangles into two groups. When the teacher asks why they sorted the pictures that way, the child explains in their home language that the round ones go together and the pointy ones go together.

■● Matching icons indicate alignment of examples across age-ranges

Foundation 4.4 **Composing Shapes**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Use two- or three-dimensional shapes to represent different elements of a picture or design (for example, adding a circle in a corner to represent the sun).	Combine different two- or three-dimensional shapes to create a picture or design (for example, make a house with two blocks shaped like rectangular prisms and one shaped like a triangular prism).

Early Examples

■ While making a collage with shape stickers, a child adds a big triangle in the middle to represent a mountain and a few star stickers in the sky to show that it is nighttime. The teacher asks the child to explain what is in the picture and what shapes they used.

● A child uses flannel pieces of different shapes to create a design for the quilt they are making with their grandma. They use a small circle to make a sun and bigger circles for the clouds.

A child creates a design by putting shape blocks together in a pattern of square-triangle-square-triangle.

Later Examples

■ While making a collage with shape stickers, a child sticks a big rectangle sticker in the middle of the paper to make the front wall. Then they stick a small rectangle and two circles on the big rectangle for the door and windows. Finally, they add a triangle at the top to make a roof. The teacher asks the child to explain what is in the picture and what shapes they used.

● A child uses flannel pieces of different shapes to create a design for the quilt they are making with their grandma. They use a circle and little rectangles to make a sun and its rays, and they put five triangles together to make stars.

A child creates a fish from different shapes using a computer program.

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■● Matching icons indicate alignment of examples across age-ranges

(continued)

Foundation 4.4 **Composing Shapes**

Early	**Later**
3 to 4½ Years	**4 to 5½ Years**

Early Examples (continued)

When making a farm out of blocks, a child puts a rectangular block in the center and describes to a peer in Arabic, “This is the farmhouse.” The peer then places small square blocks around the rectangle and replies, “These are the cows.”

Later Examples (continued)

For a Diwali* celebration, a child makes rangoli** with the help of a teacher. They start by drawing a circle in the middle and add triangles, squares, and lines in symmetrical patterns around the circle.

*Diwali is the festival of lights, celebrated in Indian religions including Hinduism, Jainism, Sikhism, and Newar Buddhism.

**Rangoli is an art form from India in which patterns are created using a variety of colored powders. Rangolis are often made during Hindu festivals such as Diwali.

Developing an Understanding of Two- and Three-Dimensional Shapes

Children can perceive the differences among various shapes from a very early age. In preschool, children learn to use key properties of a shape (for example, the number of corners or sides) to identify a shape. They also learn to combine shapes to make new shapes, patterns, and designs. In addition to using an evidence-based mathematics curriculum with playful activities that follow a specific scope and sequence, teachers can support children in developing an understanding of two- and three-dimensional shapes during everyday interactions and routines by:

- Providing open-ended materials that allow children to explore, sort, and build with a variety of two- and three-dimensional shapes. These materials can include blocks and tangrams as well as everyday items such as containers and cardboard boxes.
- Referring to shapes and encouraging the use of two- and three-dimensional shape names in everyday interactions.
- Inviting children to notice different representations of shapes in their environment, for example, going on a Shape Scavenger Hunt and finding examples of circles.
- Asking questions to encourage children to compare shapes and discuss their properties, for example, “How do you know this shape is a triangle?” “How are these two shapes different?” “How are they the same?”
- Presenting varied examples of each shape category, including less typical versions of a shape (for example, a scalene triangle, a square presented with the point down). For children to learn about the defining properties of a shape, they need to be exposed to many different versions of that shape. This will help them understand why one shape is a square and another is a rectangle, despite these two shapes having similar properties.
- Inviting children to create shapes in a variety of ways and compose shapes into new shapes, pictures, or designs. For example, teachers can ask, “How can we use these triangles to create a square?” “Can you make a flower using these shapes?”

Sub-Strand — Spatial Thinking

Foundation 4.5 **Positions and Directions in Space**

Early 3 to 4 ½ Years	Later 4 to 5 ½ Years
Identify some positions of objects and people in space such as in/on, under/over, up/down, and inside/outside.	Identify positions of objects and people in space including in/on, under/over, up/down, inside/outside, near/far, next to, beside/between, and in front of/behind.

Languages differ in what words they use to describe positions and locations in space. For example, Spanish uses "*en*" to describe locations that in English would be described with either "in" or "on," and Korean uses verbs such as "*kkita*" to describe something with a tight fit and several other verbs to describe looser fits, all of which would be called "in" in English. The language or languages a child learns will influence how they think about and describe position and location. For example, when communicating in English, a child whose home language is Spanish might say "in the table" instead of "on the table."

Early Examples

■ During a scavenger hunt, the teacher asks a child to look around the classroom and find who is sitting on the bench. The child finds a teddy bear sitting on the bench and brings it back to the teacher.

● A child communicates, "Where's my book?" A peer points to the table and says, "On the table."

A child goes under the table when the teacher asks, "Would you please pick up the cup? It's under the table."

Later Examples

■ During a scavenger hunt, the teacher asks a child to look around the classroom and find who is sitting between the bookshelf and the table. The child finds a teddy bear sitting between the bookshelf and the table and brings it back to the teacher.

● A child communicates, "Where's my book?" A peer says, "It's over there, next to the blocks." The child finds the book.

A child follows the teacher's directions when working on weaving a Native basket by taking willow and going over and under with their hands.

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■● Matching icons indicate alignment of examples across age-ranges

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Foundation 4.5 **Positions and Directions in Space**

Early **3 to 4½ Years**	**Later** **4 to 5½ Years**

Early Examples (continued)

When playing in the playhouse, a child tells a peer in Mandarin, “Put the *jiaozi** inside the pot.”

A child looks up when the teacher says, “If you look up, you’ll see your coat.”

Later Examples (continued)

A child follows directions when asked by the teacher to stand behind another child.

A child follows directions for an obstacle course by going under the table, crawling through the tunnel, and hopping across the rug.

*A *jiaozi* is a type of Chinese dumpling that consists of a sweet or savory filling wrapped in dough.

History–Social Science and **Foundational Language Development** — The above foundation is related to History–Social Science foundation 5.2 on communicating locations and directions and to foundation 1.3 in Foundational Language Development on understanding and using size and location words. Domain-relevant foundations that pertain to describing position and directionality have been intentionally included in all three domains. In Mathematics, this foundation is included to highlight children’s understanding of spatial concepts and their ability to communicate about spatial positions, locations, and directions.

Foundation 4.6 **Mental Rotation**

Early **3 to 4 ½ Years**	**Later** **4 to 5 ½ Years**
Rely on trial and error to determine how objects move in space and fit in different locations (for example, try to fit an object into a hole by rotating, flipping, or sliding the piece in different orientations until it fits).	Rotate, flip, or slide objects to solve a problem without relying as much on physical trial and error (for example, rotate an object before fitting it into a hole).

Early Examples

■ When putting together a wooden puzzle, a child slides each piece around until it fits into the correct place.

● A child builds a castle using magnet tiles, but struggles to connect two of the pieces. The teacher picks up a separate piece and says, “Sometimes when I am stuck, I try to turn the piece until it fits” and models rotating their own tile piece. The child then turns their tile piece until it aligns and connects with the other tile.

When trying to move a stool from one side of the room to the other, a child attempts to slide it through the gap between the table and the bookcase. After the child realizes the space is not big enough for the stool to fit through, they push it to the other side of the table where there is more space.

Later Examples

■ When putting together a puzzle with a peer, a child picks up a puzzle piece and communicates to their peer, “This one goes here.” The child then turns the puzzle piece before fitting it into the correct place.

● A child builds a castle using magnet tiles. The child rotates a triangular tile so that the point is down and puts that triangle between two other triangular tiles that have the point up. The child continues to add to the pattern of alternating triangles with point up and point down, rotating each tile to fit.

A child with cerebral palsy plays a puzzle game on the computer. They use the arrow keys to turn shapes and then fit them into place. When a new shape appears, the teacher asks, “What do you need to do to make it fit?” The child flips the piece once and moves it to the right spot.

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■ ● Matching icons indicate alignment of examples across age-ranges

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Foundation 4.6 **Mental Rotation**

Early	Later
3 to 4½ Years	**4 to 5½ Years**

Early Examples (continued)

While draping a sari,* a child flips over the material so that the decorated side is facing the front. The teacher says, "This is a beautiful sari. Do you need some help? What should we do next?" The child replies, "I want it to go around." With the help of the teacher, they wrap the material around themself.

Later Examples (continued)

When playing supermarket in the dramatic play area, a child tries to fit a toy carton of milk inside the grocery basket, but it falls out because the basket is too full. The child then removes all the items from the basket, puts the milk carton in first, and squeezes the other items into the space that remains.

*A sari is a type of clothing from South Asia.

Glossary

attribute. A property or characteristic of an object or a person. Attributes such as size, color, and shape would be apparent to a prekindergarten child and would be used in grouping and sorting.

base ten. The number system used to assign place value to numbers. The base-ten system uses digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to represent all other numbers in its system (for example, the number 32 represents three tens and two ones).

cardinality. The concept that the number name applied to the last object counted represents the total number of objects in the group (the quantity of objects counted).

classification. The sorting, grouping, or categorizing of objects according to established criteria.

mental rotation. The ability to imagine what an object would look like when rotated or seen from a different angle.

numeral. The symbol used to express a number.

one-to-one correspondence. One and only one number word is used for each object in the array of objects being counted.

operations. The process of calculating a value using addition, subtraction, multiplication, or division.

pattern. A sequence of objects, pictures, or numbers that repeat based on a specific rule.

place value. The value of where a digit is in a number. (For example, in the number 427, the 2 is in the tens place, so its place value is 10.)

subitize. The ability to quickly and accurately determine the quantity of objects in a small group (of up to five objects) without actually counting the objects.

three-dimensional shape. A solid figure that has three dimensions: length, width, and height.

two-dimensional shape. A flat shape that has two dimensions: length and width.

References and Source Materials

Ashkenazi, S., H. Haber, V. Shemesh, and S. Silverman. 2022. "Early Subitizing Development: The Role of Visuospatial Working Memory." *European Journal of Education and Pedagogy* 3 (2): 79–85.

Atinuke. 2017. *Baby Goes to Market*. Somerville, MA: Candlewick Press.

Baroody, A. J., and M. Lai. 2022. "The Development and Assessment of Counting-Based Cardinal Number Concepts." *Educational Studies in Mathematics* 111 (2): 185–205.

Baroody, A. J., M. Lai, and K. S. Mix. 2017. "Assessing Early Cardinal-Number Concepts." *In Proceedings of the 39th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, edited by E. Galindo and J. Newton. Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Barth, H., K. La Mont, J. Lipton, and E. S. Spelke. 2005. "Abstract Number and Arithmetic in Preschool Children." In *Proceedings of the National Academy of Sciences* 102 (39): 14116–14121.

Benoit, L., H. Lehalle, and F. Jouen. 2004. "Do Young Children Acquire Number Words Through Subitizing or Counting?" *Cognitive Development* 19 (3): 291–307.

Blair, C., and R. P. Razza. 2007. "Relating Effortful Control, Executive Function, and False Belief Understanding to Emerging Math and Literacy Ability in Kindergarten." *Child Development* 78 (2): 647–663.

Blevins-Knabe, B. 2016. "Early Mathematical Development: How the Home Environment Matters." In *Early Childhood Mathematics Skill Development in the Home Environment*, edited by B. Blevins-Knabe and A. M. Berghout Austin. Switzerland: Springer International Publishing.

California Department of Education. 2023. *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve*. Sacramento, CA: California Department of Education.

Cankaya, O., J. LeFevre. 2016. "The Home Numeracy Environment: What Do Cross-Cultural Comparisons Tell Us About How to Scaffold Young Children's Mathematical Skills?" In *Early Childhood Mathematics Skill Development in the Home Environment*, edited by B. Blevins-Knabe and A. M. Berghout Austin. Switzerland: Springer International Publishing.

Cankaya, O., J. LeFevre, and K. Dunbar. 2014. "The Role of Number Naming Systems and Numeracy Experiences in Children's Rote Counting: Evidence from Turkish and Canadian Children." *Learning and Individual Differences* 32:238–245.

Chernyak, N., P. L. Harris, and S. Cordes. 2022. "A Counting Intervention Promotes Fair Sharing in Preschoolers." *Child Development* 93 (5): 1365–1379.

Chernyak, N., B. Sandham, P. L. Harris, and S. Cordes. 2016. "Numerical Cognition Explains Age-Related Changes in Third-Party Fairness." *Developmental Psychology* 52 (10): 1555.

Cheung, P., M. Rubenson, and D. Barner. 2017. "To Infinity and Beyond: Children Generalize the Successor Function to All Possible Numbers Years After Learning to Count." *Cognitive Psychology* 92:22–36.

Claessens, A., and M. Engel. 2013. "How Important Is Where You Start? Early Mathematics Knowledge and Later School Success." *Teachers College Record* 115 (6): 1–29.

Clark, C. A. C., V. E. Pritchard, and L. J. Woodward. 2010. "Preschool Executive Functioning Abilities Predict Early Mathematics Achievement." *Developmental Psychology* 46 (5): 1176–1191.

Clements, D. H. 1999. "Subitizing: What Is It? Why Teach It?" *Teaching Children Mathematics* 5 (7): 400–405.

Clements, D. H., and J. Sarama. 2000. "Young Children's Ideas About Geometric Shapes." *Teaching Children Mathematics* 6 (8): 482–488.

Clements, D. H., and J. Sarama. 2015. "Discussion from a Mathematics Education Perspective." *Mathematical Thinking and Learning* 17 (2–3): 244–252.

Clements, D. H., and J. Sarama. 2021. *Learning and Teaching Early Math: The Learning Trajectories Approach*. 3rd ed. New York, NY: Routledge.

Clements, D. H., S. Swaminathan, M. A. Z. Hannibal, and J. Sarama. 1999. "Young Children's Concepts of Shape." *Journal for Research in Mathematics Education* 30 (2): 192–212.

Dillon, M. R., V. Izard, and E. S. Spelke. 2020. "Infants' Sensitivity to Shape Changes in 2D Visual Forms." *Infancy* 25 (5): 618–639.

Duncan, G. J., C. J. Dowsett, A. Claessens, K. Magnuson, A. C. Huston, P. Klebanov, L. S. Pagani, L. Feinstein, M. Engel, J. Brooks-Gunn, H. Sexton, K. Duckworth, and C. Japel. 2007. "School Readiness and Later Achievement." *Developmental Psychology* 43 (6): 1428.

Fox, J. 2005. "Child-Initiated Mathematical Patterning in the Pre-Compulsory Years." In *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* 2:313–320.

Frick, A., M. A. Hansen, and N. S. Newcombe. 2013. “Development of Mental Rotation in 3- to 5-Year-Old Children.” *Cognitive Development* 28 (4): 386–399.

Frick, A., and N. S. Newcombe. 2012. “Getting the Big Picture: Development of Spatial Scaling Abilities.” *Cognitive Development* 27 (3): 270–282.

Gal, H., and L. Linchevski. 2010. “To See or Not to See: Analyzing Difficulties in Geometry from the Perspective of Visual Perception.” *Educational Studies in Mathematics* 74 (2): 163–183.

Galindo, C., S. Sonnenschein, and A. Montoya-Ávila. 2019. “Latina Mothers’ Engagement in Children’s Math Learning in the Early School Years: Conceptions of Math and Socialization Practices.” *Early Childhood Research Quarterly* 47:271–283.

García, O., and L. Wei. 2014. *Translanguaging: Language, Bilingualism and Education*. New York, NY: Palgrave Macmillan.

Ginsburg, H. P. 2006. “Mathematical Play and Playful Mathematics: A Guide for Early Education. In P*lay = Learning: How Play Motivates and Enhances Children’s Cognitive and Social–Emotional Growth*, edited by D. G. Singer, R. M. Golinkoff, and K. Hirsh-Pasek. New York, NY: Oxford University Press.

Gundersheimer, Ben. 2019. *Señorita Mariposa*. New York, NY: Nancy Paulsen Books.

Gunderson, E. A., and S. C. Levine. 2011. “Some Types of Parent Number Talk Count More Than Others: Relations Between Parents’ Input and Children’s Cardinal-Number Knowledge.” *Developmental Science* 14 (5): 1021–1032.

Hawes, Z., D. Tepylo, and J. Moss. 2015. “Developing Spatial Thinking.” In *Spatial Reasoning in the Early Years: Principles, Assertions, and Speculations*, edited by B. Davis and the Spatial Reasoning Study Group. New York, NY: Routledge.

Huttenlocher, J., M. Vasilyeva, N. Newcombe, and S. Duffy. 2008. “Developing Symbolic Capacity One Step at a Time.” *Cognition* 106 (1): 1–12.

Johnson, N. C., A. C. Turrou, B. G. McMillan, M. C. Raygoza, and M. L. Franke. 2019. “‘Can You Help Me Count These Pennies?’: Surfacing Preschoolers’ Understandings of Counting.” *Mathematical Thinking and Learning* 21 (4): 237–264.

Jordan, N. C., D. Kaplan, C. Ramineni, and M. N. Locuniak. 2009. “Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes.” *Developmental Psychology* 45 (3): 850.

Knudsen, B., M. Fischer, A. Henning, and G. Aschersleben. 2015. “The Development of Arabic Digit Knowledge in 4- to 7-Year-Old Children.” *Journal of Numerical Cognition* 1 (1): 21–37.

Langhorst, P., A. Ehlert, and A. Fritz. 2012. "Non-numerical and Numerical Understanding of the Part–Whole Concept of Children Aged 4 to 8 in Word Problems." *Journal für Mathematik-Didaktik* 33 (2): 233–262.

Lehrl, S., K. Kluczniok, and H. G. Rossbach. 2016. "Longer-Term Associations of Preschool Education: The Predictive Role of Preschool Quality for the Development of Mathematical Skills Through Elementary School." *Early Childhood Research Quarterly* 36:475–488.

Levine, S. C., L. W. Suriyakham, M. L. Rowe, J. Huttenlocher, and E. A. Gunderson. 2010. "What Counts in the Development of Young Children's Number Knowledge?" *Developmental Psychology* 46 (5): 1309.

MacDonald, B. L., and J. L. Wilkins. 2019. "Subitising Activity Relative to Units Construction: A Case Study." *Research in Mathematics Educatio*n 21 (1): 77–95.

Mark, W., and A. Dowker. 2015. "Linguistic Influence on Mathematical Development Is Specific Rather Than Pervasive: Revisiting the Chinese Number Advantage in Chinese and English Children." *Frontiers in Psychology* 6:203.

Melhuish, E. C., M. B. Phan, K. Sylva, P. Sammons, I. Siraj-Blatchford, and B. Taggart. 2008. "Effects of the Home Learning Environment and Preschool Center Experience upon Literacy and Numeracy Development in Early Primary School." *Journal of Social Issues* 64 (1): 95–114.

Melhuish, E. C., K. Sylva, P. Sammons, I. Siraj-Blatchford, B. Taggart, M. B. Phan, and A. Malin. 2008. "Preschool Influences on Mathematics Achievement." *Science* 321 (5893): 1161–1162.

Muldoon, K. P., C. Lewis, and B. Francis. 2007. "Using Cardinality to Compare Quantities: The Role of Social–Cognitive Conflict in Early Numeracy." *Developmental Science* 10 (5): 694–711.

Navajo People. 2023. Navajo Rugs. https://navajopeople.org/navajo-rugs.htm.

Nguyen, T., T. W. Watts, G. J. Duncan, D. H. Clements, J. S. Sarama, C. Wolfe, and M. E. Spitler. 2016. "Which Preschool Mathematics Competencies Are Most Predictive of Fifth Grade Achievement?" *Early Childhood Research Quarterly* 36:550–560.

Öcal, T., and M. Halmatov. 2021. "3D Geometric Thinking Skills of Preschool Children." *International Journal of Curriculum and Instruction* 13 (2): 1508–1526.

Pagliaro, C. M., and K. L. Kritzer. 2013. "The Math Gap: A Description of the Mathematics Performance of Preschool-Aged Deaf/Hard-of-Hearing Children." *The Journal of Deaf Studies and Deaf Education* 18 (2): 139–160.

Paik, J. H., L. van Gelderen, M. Gonzales, P. F. de Jong, and M. Hayes. 2011. “Cultural Differences in Early Math Skills Among U.S., Taiwanese, Dutch, and Peruvian Preschoolers.” *International Journal of Early Years Education* 19 (2): 133–143.

Pixner, S., V. Dresen, and K. Moeller. 2018. “Differential Development of Children’s Understanding of the Cardinality of Small Numbers and Zero.” *Frontiers in Psychology* 9:1636.

Pruden, S. M., S. C. Levine, and J. Huttenlocher. 2011. “Children’s Spatial Thinking: Does Talk About the Spatial World Matter?” *Developmental Science* 14 (6): 1417–30.

Purpura, D. J., S. A. Schmitt, and C. M. Ganley. 2017. “Foundations of Mathematics and Literacy: The Role of Executive Functioning Components.” *Journal of Experimental Child Psychology* 153:15–34.

Ringgold, F. 2004. *Cassie’s Word Quilt*. New York, NY: Alfred A. Knopf.

Rittle-Johnson, B., E. R. Fyfe, L. E. McLean, and K. L. McEldoon. 2013. “Emerging Understanding of Patterning in 4-Year-Olds.” *Journal of Cognition and Development* 14 (3): 376–396.

Santos, S., and S. Cordes. 2022. “Math Abilities in Deaf and Hard of Hearing Children: The Role of Language in Developing Number Concepts.” *Psychological Review* 129 (1): 199–211.

Sarnecka, B. W., and S. Carey. 2008. “How Counting Represents Number: What Children Must Learn and When They Learn It.” *Cognition* 108 (3): 662–674.

Sarnecka, B. W., J. Negen, and M. C. Goldman. 2018. “Early Number Knowledge in Dual-Language Learners from Low-SES Households.” In *Language and Culture in Mathematical Cognition*, edited by D. B. Berch, D. C. Geary, and K. M. Koepke. San Diego, CA: Elsevier Academic Press.

Sarnecka, B. W., and C. E. Wright. 2013. “The Idea of an Exact Number: Children’s Understanding of Cardinality and Equinumerosity.” *Cognitive Science* 37 (8): 1493–1506.

Shusterman, A., and P. Li. 2016. “Frames of Reference in Spatial Language Acquisition.” *Cognitive Psychology* 88:115–161.

Sinclair, N., and J. Moss. 2012. “The More It Changes, the More It Becomes the Same: The Development of the Routine of Shape Identification in Dynamic Geometry Environment.” *International Journal of Educational Research* 51–52:28–44.

Smidts, D. P., R. Jacobs, and V. Anderson. 2004. “The Object Classification Task for Children (OCTC): A Measure of Concept Generation and Mental Flexibility in Early Childhood.” *Developmental Neuropsychology* 26 (1): 385–401.

Starkey, G. S., and B. D. McCandliss. 2014. "The Emergence of 'Groupitizing' in Children's Numerical Cognition." *Journal of Experimental Child Psychology* 126:120–137.

Stephan, M., and D. H. Clements. 2003. "Linear and Area Measurement in Prekindergarten to Grade 2." In *Learning and Teaching Measurement*. Reston, VA: National Council of Teachers of Mathematics.

Szilágyi, J., D. H. Clements, and J. Sarama. 2013. "Young Children's Understandings of Length Measurement: Evaluating a Learning Trajectory." *Journal for Research in Mathematics Education* 44 (3): 581–620.

Throndsen, J., B. MacDonald, and J. Hunt. 2017. "Developing a Kindergartener's Concept of Cardinality." *Australian Primary Mathematics Classroom* 22 (2): 21–25.

Tsamir, P., D. Tirosh, E. S. Levenson, R. Barkai, and M. Tabach. 2017. "Repeating Patterns in Kindergarten: Findings from Children's Enactments of Two Activities." *Educational Studies in Mathematics* 96 (1): 83–99.

Verdine, B. N., R. M. Golinkoff, K. Hirsh-Pasek, and N. Newcombe. 2017. *Links Between Spatial and Mathematical Skills Across the Preschool Years*. Hoboken, NJ: Wiley.

Verdine, B. N., K. R. Lucca, R. M. Golinkoff, K. Hirsh-Pasek, and N. S. Newcombe. 2016. "The Shape of Things: The Origin of Young Children's Knowledge of the Names and Properties of Geometric Forms." *Journal of Cognition and Development* 17 (1): 142–161.

Wagner, K., K. Kimura, P. Cheung, and D. Barner. 2015. "Why Is Number Word Learning Hard? Evidence from Bilingual Learners." *Cognitive Psychology* 83:1–21.

Wijns, N., J. Torbeyns, M. Bakker, B. De Smedt, and L. Verschaffel. 2019. "Four-Year Olds' Understanding of Repeating and Growing Patterns and Its Association with Early Numerical Ability." *Early Childhood Research Quarterly* 49:152–163.

Wynn, K. 1992. "Children's Acquisition of the Number Words and the Counting System." *Cognitive Psychology* 24 (2): 220–251.

Xu, F., and E. S. Spelke. 2000. "Large Number Discrimination in 6-Month-Old Infants." *Cognition* 74 (1): B1–B11.

Yun, C., A. Havard, D. C. Farran, M. W. Lipsey, C. Bilbrey, and K. G. Hofer. 2011. "Subitizing and Mathematics Performance in Early Childhood." In *Proceedings of the Annual Meeting of the Cognitive Science Society* 33.

Zhang, X., C. Chen, T. Yang, and X. Xu. 2020. "Spatial Skills Associated with Block-Building Complexity in Preschoolers." *Frontiers in Psychology* 11:563493.

Zosh, J. M., E. J. Hopkins, H. Jensen, C. Liu, D. Neale, K. Hirsh-Pasek, S. L. Solis, and D. Whitebread. 2017. *Learning Through Play: A Review of the Evidence*. Billund, Denmark: LEGO Foundation.